Dipole-mode wakefields in dielectric-loaded rectangular waveguide accelerating structures

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(Received 19 November 2002; published 21 July 2003)

By applying different symmetric boundary conditions, we found that the transverse wakefields generated by an electron bunch traveling through a partially loaded rectangular dielectric structure at an off center position can be decomposed into corresponding orthogonal longitudinal section electric (LSE) and longitudinal section magnetic (LSM) modes for guided waves as in the case of longitudinal wakefields treated previously. The wakefields are characterized using the normalized shunt impedance R/Q, a function of the geometry of the accelerating structure, for both LSE and LSM modes. A numerical example is given for an X-band waveguide structure and detailed results are given for the several leading transverse wakefield terms. The analytic results obtained are in agreement with the results from the time domain simulation tool MAFIA®.

DOI: 10.1103/PhysRevE.68.016502 PACS number(s): 41.75.Lx, 29.17.+w, 42.82.Et, 84.40.−x

In our previous paper [1], we have given a detailed description of wakefields generated by a bunched relativistic beam passing through the center of a partially dielectric loaded rectangular waveguide structure. However, a complete treatment should also include transverse (dipole) wakefield effects since these fields can cause severe beam instabilities. Previous treatments [2,3] of the transverse wakefield effects in similar structures did not include all cases and used a complicated mathematical formalism. In this paper, we extend our previous work to include transverse wakefields excited by a beam traveling with offsets in both the X (horizontal) and the Y (vertical) directions. We will show that the transverse wakefields in this type structure can be calculated using both the transverse resonance and the mode matching methods [4] which are used to construct the X and Y dipole modes. Rather than solving the bunch-excited fields through Maxwell's equations directly, the wakefields can be expressed as a sum over waveguide parameter R/Q, or equivalent of loss factor in most literatures (a function of the structure geometry only) for each mode. The summation is an infinite series and it can be truncated by the finite bunch length of the drive beam, as in case of most wakefields calculations. One should point out that a similar method [5] was used to find loss factors and wakefields of metal periodic rectangular structures. In this paper, we first give an analytical theory and then a numerical example for an X-band dielectric-loaded accelerating structure, and finally a comparison with results from the commercial wakefield simulation code MAFIA® is given to check the validity of our calculations.

As can be inferred from the geometry shown in Fig. 1, the intrinsic modes in a dielectric-loaded rectangular waveguide are either LSM (longitudinal section magnetic) or LSE (longitudinal section electric) modes that, respectively, have no H or E components normal to the vacuum-dielectric interface. This corresponds to assuming the transverse direction to the interface normal vector to be the direction of propagation. A short electron bunch traveling through dielectric-loaded rectangular waveguide at a transverse offset (x_0,y_0) will excite both x- and y-dipole modes in which x-dipole mode can be expressed as superposition of $\mathrm{LSM}_{mn}^{(\mathrm{open})}$ and $\mathrm{LSE}_{mn}^{(\mathrm{open})}$ $(m=2,4,6,\cdots)$ modes and y-dipole as $\mathrm{LSM}_{mn}^{(\mathrm{short})}$ and $\mathrm{LSE}_{mn}^{(\mathrm{short})}$ $(m=1,3,5,\cdots)$ modes. Here, the superscript (short) or (open) refers to short $(E_\parallel=0)$ or open symmetric boundary conditions at the midplane of the rectangular waveguide as shown in Fig. 1.

Computation of dipole modes by the summation of the corresponding LSE and LSM modes is based on the relation between system response and signal excitation. The waveguide considered here is a passive system, and the traveling electron bunch is equivalent to be a point particle with $j_z=ev\,\delta(z-vt)\,\delta(x-x_0)\,\delta(y-y_0)$.

Following a similar procedure to that of Ref. [1], we can obtain the dispersion relations of the open and short central plane cases, respectively, through the transverse resonance method [4] as follows:

for open central plane case

$$-Z_{0mn}^{(0)}\cot(k_{ymn}^{(0)}a)+Z_{0mn}^{(1)}\tan[k_{ymn}^{(1)}(b-a)]=0, \tag{1a}$$

for short central plane case

$$Z_{0mn}^{(0)}\tan(k_{ymn}^{(0)}a)+Z_{0mn}^{(1)}\tan[k_{ymn}^{(1)}(b-a)]=0, \tag{1b}$$

where

$$Z_{0mn}^{(0)}=\frac{k_{ymn}^{(0)}}{\omega\varepsilon_0},\quad Z_{0mn}^{(1)}=\frac{k_{ymn}^{(1)}}{\omega\varepsilon_0\varepsilon_s}\quad(\mathrm{LSM}_{mn})$$

and

$$Z_{0mn}^{(0)}=\frac{\omega\mu_0}{k_{ymn}^{(0)}},\quad Z_{0mn}^{(1)}=\frac{\omega\mu_0}{k_{ymn}^{(1)}}\quad(\mathrm{LSE}_{mn}),$$

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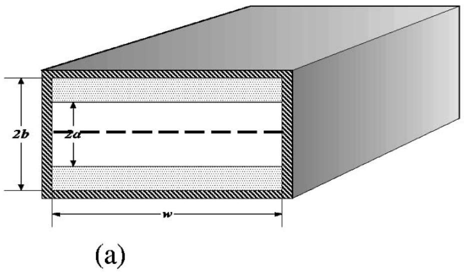

FIG. 1. Dielectric-loaded rectangular guide with symmetrical boundary condition; (a) open plane case for x-dipole mode computation, and (b) short plane case for y-dipole mode computation.

are the values of the characteristic impedances of each mode. The transverse propagation constants are expressed in terms of the longitudinal propagation constant β_{mn} using the following conditions:

$$k_{ymn}^{(0)^2}=k_0^2-\left(\frac{m\pi}{w}\right)^2-\beta_{mn}^2 \tag{2a}$$

and

$$k_{ymn}^{(1)^2}=\varepsilon_r k_0^2-\left(\frac{m\pi}{w}\right)^2-\beta_{mn}^2, \tag{2b}$$

where $k_0=2\pi f/c$ is the propagation constant in free space and the superscript (0) or (1) refers to the vacuum or dielectric region of the waveguide, respectively.

The field components of the LSM$^{(\text{open})}$, LSE$^{(\text{open})}$, LSM$^{(\text{short})}$, and LSE$^{(\text{short})}$ modes in a dielectric-loaded rectangular waveguide can be derived from the solution of vector potential wave equations:

$$\text{LSM mode:}\quad \vec{\psi}_e=\vec{\alpha}_y\psi_e(x,y,z)$$

$$E_x=-j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2\psi_e}{\partial x\,\partial y},\quad H_x=-\frac{1}{\mu}\frac{\partial\psi_e}{\partial z},$$

$$E_y=-j\frac{1}{\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial y^2}+\beta^2\right)\psi_e,\quad H_y=0,$$

$$E_z=-j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2\psi_e}{\partial y\partial z},\quad H_z=\frac{1}{\mu}\frac{\partial\psi_e}{\partial x}, \tag{3a}$$

$$\text{LSE mode:}\quad \vec{\psi}_h=\vec{\alpha}_y\psi_h(x,y,z)$$

$$E_x=\frac{1}{\varepsilon}\frac{\partial\psi_h}{\partial z},\quad H_x=-j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2\psi_h}{\partial x\,\partial y},$$

$$E_y=0,\quad H_y=-j\frac{1}{\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial y^2}+\beta^2\right)\psi_h,$$

$$E_z=-\frac{1}{\varepsilon}\frac{\partial\psi_h}{\partial x},\quad H_z=-j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2\psi_h}{\partial y\,\partial z}, \tag{3b}$$

where ψ_e and ψ_h must satisfy the scalar wave equations

$$\nabla^2\psi_q(x,y,z)+\beta^2\psi_q(x,y,z)=0,\quad q=e,h. \tag{4}$$

Applying the boundary conditions at the perfectly conducting guide walls ($x=\pm w/2$, $y=b$) and the boundary condition at the magnetic or electric wall ($y=0$), the potential functions ψ_e for open short modes are

$$\psi_{emn}^{(\text{open})}=\begin{cases} A_{mn}^{(\text{open})}\sin\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\sin k_{ymn}^{(0)}y e^{-j\beta_{mn}z}, & 0<y<a\\ B_{mn}^{(\text{open})}\sin\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\cos k_{ymn}^{(1)}(b-y)e^{-j\beta_{mn}z}, & a<y<b\end{cases} \tag{5a}$$

and

$$\psi_{emn}^{(\text{short})}=\begin{cases} A_{mn}^{(\text{short})}\sin\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\cos k_{ymn}^{(0)}y e^{-j\beta_{mn}z}, & 0<y<a,\\ B_{mn}^{(\text{short})}\sin\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\cos k_{ymn}^{(1)}(b-y)e^{-j\beta_{mn}z}, & a<y<b,\end{cases} \tag{5b}$$

where

$$\frac{A_{mn}^{(\text{open})}}{B_{mn}^{(\text{open})}}=\frac{\varepsilon_r\cos k_{ymn}^{(1)}(b-a)}{\sin k_{ymn}^{(0)}a}\quad\text{and}\quad\frac{A_{mn}^{(\text{short})}}{B_{mn}^{(\text{short})}}=\frac{\varepsilon_r\cos k_{ymn}^{(1)}(b-a)}{\cos k_{ymn}^{(0)}a}.$$

In each region, the fields for the LSE$_{mn}^{(\text{open})}$ or LSE$_{mn}^{(\text{short})}$ are derived from a magnetic-type potential function. Using the boundary conditions at the conducting guide walls ($x=\pm w/2$, $y=b$) and the boundary condition at the magnetic or electric wall ($y=0$), the potential functions ψ_h are

$$\psi_{hmn}^{(\text{open})} = \begin{cases} C_{mn}^{(\text{open})} \dfrac{1}{j\omega\mu_0} \cos\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\cos k_{ymn}^{(0)} y e^{-j\beta_{mn}z}, & 0<y<a \\ D_{mn}^{(\text{open})} \dfrac{1}{j\omega\mu_0} \cos\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\sin k_{ymn}^{(1)} (b-y) e^{-j\beta_{mn}z}, & a<y<b \end{cases} \tag{6a}$$

and

$$\psi_{hmn}^{(\text{short})} = \begin{cases} C_{mn}^{(\text{short})} \dfrac{1}{j\omega\mu_0} \cos\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\sin k_{ymn}^{(0)} y e^{-j\beta_{mn}z}, & 0<y<a, \\ D_{mn}^{(\text{short})} \dfrac{1}{j\omega\mu_0} \cos\dfrac{m\pi}{w}\left(x+\dfrac{w}{2}\right)\sin k_{ymn}^{(1)} (b-y) e^{-j\beta_{mn}z}, & a<y<b, \end{cases} \tag{6b}$$

where

$$\frac{C_{mn}^{(\text{open})}}{D_{mn}^{(\text{open})}} = \frac{\sin k_{ymn}^{(1)}(b-a)}{\cos k_{ymn}^{(0)} a} \quad \text{and} \quad \frac{C_{mn}^{(\text{short})}}{D_{mn}^{(\text{short})}} = \frac{\sin k_{ymn}^{(1)}(b-a)}{\sin k_{ymn}^{(0)} a}.$$

Substituting these potential functions into Eq. (3), the z component of the electric field in the vacuum region can be written as follows:

$$E_{z,\text{LSM}}^{(\text{open})}(x,y,z) = A_{mn}^{(\text{open})}(-\beta_{mn}) k_{y,mn}^{(0)} \sin\frac{m\pi}{w}\left(x+\frac{w}{2}\right)\cos k_{y,mn}^{(0)} y \exp(-j\beta z), \tag{7a}$$

$$E_{z,\text{LSE}}^{(\text{open})}(x,y,z) = C_{mn}^{(\text{open})}(-j)\frac{m\pi}{w}\sin\frac{m\pi}{w}\left(x+\frac{w}{2}\right)\cos k_{y,mn}^{(0)} y \exp(-j\beta z), \tag{7b}$$

$$E_{z,\text{LSM}}^{(\text{short})}(x,y,z) = A_{mn}^{(\text{short})}\beta_{mn} k_{y,mn}^{(0)} \sin\frac{m\pi}{w}\left(x+\frac{w}{2}\right)\sin k_{y,mn}^{(0)} y \exp(-j\beta z), \tag{7c}$$

$$E_{z,\text{LSE}}^{(\text{short})}(x,y,z) = C_{mn}^{(\text{short})}(-j)\frac{m\pi}{w}\sin\frac{m\pi}{w}\left(x+\frac{w}{2}\right)\sin k_{y,mn}^{(0)} y \exp(-j\beta z). \tag{7d}$$

According to the characteristics of intrinsic guided wave modes that generate the corresponding transverse forces on the traveling electron, the x-dipole modes are a linear superposition of $\text{LSM}_{(2k)n}^{(\text{open})}$ and $\text{LSE}_{(2k)n}^{(\text{open})}$, and the y-dipole modes of $\text{LSM}_{(2k-1)n}^{(\text{short})}$ and $\text{LSM}_{(2k)n}^{(\text{short})}$, where k and n are positive integers. The longitudinal electric field of the synchronous x-dipole [$\beta=2\pi f/c$, which implies $k_y^{(0)}=-j(2k\pi/w)$] and y-dipole modes [$k_y^{(0)}=-j((2k-1)\pi/w)$] can be expressed as

$$E_z^{(\text{dipole})}(x,y,z) = \begin{cases} \displaystyle\sum_{k,n}\begin{Bmatrix} A_{(2k)n}^{(\text{open})}\beta_{(2k)n}k_{y,(2k)n}^{(0)} \\ jC_{(2k)n}^{(\text{open})}\dfrac{(2k)\pi}{w}\end{Bmatrix}\sin\frac{(2k)\pi}{w}x\cos k_y^{(0)} y \exp(-j\beta_{(2k)n}z), & x \text{ dipole}, \\ \displaystyle\sum_{k,n}\begin{Bmatrix} A_{(2k-1)n}^{(\text{short})}\beta_{(2k-1)n}k_{y,(2k-1)n}^{(0)} \\ (-j)C_{(2k\text{-}1)n}^{(\text{short})}\dfrac{(2k-1)\pi}{w}\end{Bmatrix}\cos\frac{(2k-1)\pi}{w}x\sin k_y^{(0)} y \exp(-j\beta_{(2k-1)n}z), & y \text{ dipole}. \end{cases} \tag{8}$$

For a traveling wave structure, one can obtain the normalized characteristic quantity R/Q for each mode by computing the stored energy U per unit length and accelerating electrical field E_z and using the definition [6,7]

$$\left[\frac{R_{x_0,y_0}}{Q}\right]_i = \left[\frac{|E_z|_{x_0,y_0}^2}{\omega U}\right]_i, \tag{9}$$

where R/Q is evaluated at location of x_0 and y_0. For simplicity, we drop the subscript notation of x_0 and y_0 for now and use the notation $|E_z|$ as the amplitude of the axial electrical field to stand for $E_z^{\text{dipole}}(x,y,z)$ in Eq. (8).

The stored energy in the structure with length L is determined by

$$U = \frac{1}{4L}\iiint_V (\varepsilon E^2 + \mu H^2)\, d\nu. \tag{10}$$

Without loss of generality, we can express the stored energy U for the i^{th} mode as

$$U=|E_z|^2 C\int\int_V\int \psi_q\, d\nu, \tag{11}$$

where C is a dimension- and mode-related constant. When the Eq. (11) is substituted into Eq. (9), the electric field amplitude will cancel out:

$$\left[\frac{R}{Q}\right]_i=\left[\frac{1}{\omega C\int\int_V\int \psi_q\, d\nu}\right]_i . \tag{12}$$

This shows that R/Q is only a function of the geometric parameters of the structure and the particular mode, and can be calculated directly through the complete electromagnetic (EM) field component expressions and dispersion relations we derived before. The wake field amplitude excited by a point charged particle traveling at x_0 and y_0 can be obtained through [6]

$$|E_z|_{(2k)n}\sin\frac{(2k)\pi}{w}x_0\cos k_y^{(0)}y_0$$
$$=2k_{l_{(2k)n}}q=\frac{q\,\omega_{(2k)n}}{2}\left(\frac{R_{x_0,y_0}}{Q}\right)_{(2k)n},\quad x \text{ dipole}$$
$$|E_z|_{(2k-1)n}\cos\frac{(2k-1)\pi}{w}x_0\sin k_y^{(0)}y_0$$
$$=2k_{l_{(2k-1)n}}q=\frac{q\,\omega_{(2k-1)n}}{2}\left(\frac{R_{x_0,y_0}}{Q}\right)_{(2k-1)n},\quad y \text{ dipole}, \tag{13a}$$

where $k_{l_i}=1/4\omega_i(R_{x_0,y_0}/Q)_i$ is a normalized loss factor for i^{th} mode, and Eq. (8) can be simplified to

$$E_z=\sum_{k,n}\begin{cases}|E_z|_{(2k)n}\sin\dfrac{(2k)\pi}{w}x\cos k_y^{(0)}y\exp(-j\beta_{(2k)n}z), & x \text{ dipole}\\[2ex] |E_z|_{(2k-1)n}\cos\dfrac{(2k-1)\pi}{w}x\sin k_y^{(0)}y\exp(-j\beta_{(2k-1)n}z), & y \text{ dipole}.\end{cases} \tag{13b}$$

As in all the wakefield calculations, once the longitudinal wakefield E_z is obtained, transverse forces can be directly calculated from E_z by using the original [8] or extended Panofsky-Wenzel theorem [9] for Cartesian coordinate systems:

$$\frac{\partial \vec{F}_\perp}{\partial z}=e\boldsymbol{\nabla}_\perp E_z . \tag{14}$$

Thus,

$$\frac{\partial \vec{F}_\perp}{\partial z}=e\sum_{k,n}\begin{cases}|E_z|_{(2k)n}\exp(-j\beta_{(2k)n}z)\left(\vec{\alpha}_x\dfrac{(2k)\pi}{w}\cos\dfrac{(2k)\pi}{w}x\cos k_y^{(0)}y-\vec{\alpha}_y k_y^{(0)}\sin\dfrac{(2k)\pi}{w}x\sin k_y^{(0)}y\right), & x \text{ dipole}\\[2ex] |E_z|_{(2k-1)n}\exp(-j\beta_{(2k-1)n}z)\left(-\vec{\alpha}_x\dfrac{(2k-1)\pi}{w}\sin\dfrac{(2k-1)\pi}{w}x\sin k_y^{(0)}y+\vec{\alpha}_y k_y^{(0)}\cos\dfrac{(2k-1)\pi}{w}x\cos k_y^{(0)}y\right),\\ y \text{ dipole}.\end{cases} \tag{15}$$

By simple integrating both sides of Eq. (15) and keeping the real part, we have

$$\vec{F}_\perp=e\sum_{k,n}\begin{cases}|E_z|_{(2k)n}\dfrac{\sin\beta_{(2k)n}}{\beta_{(2k)n}}\left(\vec{\alpha}_x\dfrac{(2k)\pi}{w}\cos\dfrac{(2k)\pi}{w}x\cosh|k_y^{(0)}|y+\vec{\alpha}_y|k_y^{(0)}|\sin\dfrac{(2k)\pi}{w}x\sinh|k_y^{(0)}|y\right), & x \text{ dipole}\\[2ex] |E_z|_{(2k-1)n}\dfrac{\sin\beta_{(2k-1)n}}{\beta_{(2k-1)n}}\left(\vec{\alpha}_x\dfrac{(2k-1)\pi}{w}\sin\dfrac{(2k-1)\pi}{w}x\sinh|k_y^{(0)}|y+\vec{\alpha}_y|k_y^{(0)}|\cos\dfrac{(2k-1)\pi}{w}x\cosh|k_y^{(0)}|y\right),\\ y \text{ dipole}.\end{cases} \tag{16}$$

From Eq. (16), the dipole mode transverse forces will vanish when the width of the rectangular waveguide approaches infinity (under the synchronous condition $k_y^{(0)}=-jk_x^{(0)}=-jm\pi/w$), in agreement with the results of Ref. [2]. In order to quantify our analysis, we show a numerical example for the structure discussed in Ref. [1], an X-band H-plane dielectric partially loaded rectangular waveguide structure with dimensions of a$=3$, $b=5$, $w=23$ mm, and relative permittivity 10.

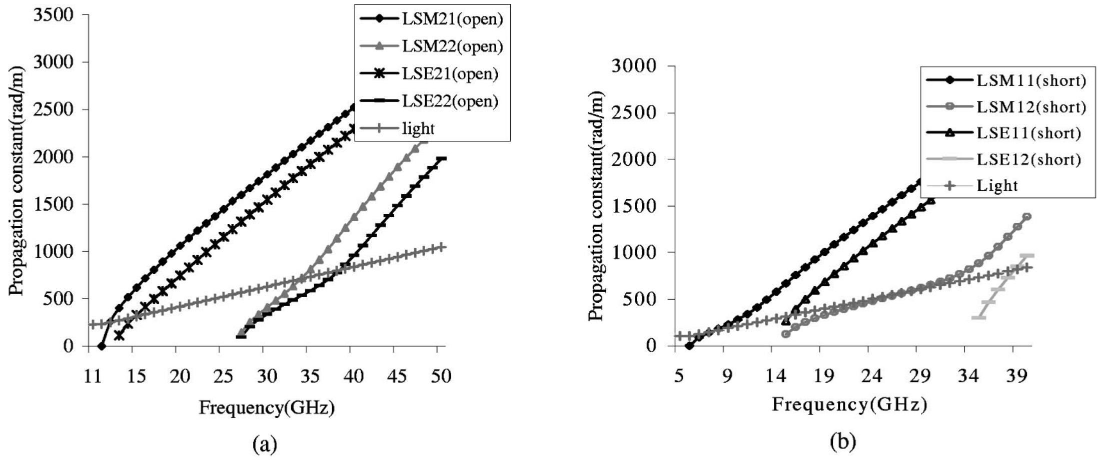

FIG. 2. Dispersion characteristics of x-dipole mode (a), and y-dipole mode (b) for the X-band waveguide described in the text.

The transcendental equation (1) is a complex function of β_{mn} and f, which gives the field components and dispersion relations of all the eigenmodes for a dielectric-loaded rectangular waveguide. For the inhomogeneous guide considered here, the dispersion relation must be solved at each frequency. Figure 2 shows the dispersion curves for several modes in this structure. The corresponding synchronous accelerating parameter for each mode is given by the intersection points between the dispersion curve of each mode and the light speed line.

Due to the finite length of wakefield structure, one has to include the group velocity effect [6]. Then, the wakefield E_z from Eq. (13b) would be

$$E_z=\sum_{k,n}\begin{cases}|E_z|_{(2k)n}\sin\dfrac{(2k)\pi}{w}x\cos k_y^{(0)}y\left(1-\dfrac{Vg_{(2k)n}}{c-Vg_{(2k)n}}\dfrac{z}{L}\right)\exp(-j\beta_{(2k)n}z), & x \text{ dipole}\\ |E_z|_{(2k-1)n}\cos\dfrac{(2k-1)\pi}{w}x\sin k_y^{(0)}y\left(1-\dfrac{Vg_{(2k-1)n}}{c-Vg_{(2k-1)n}}\dfrac{z}{L}\right)\exp(-j\beta_{(2k-1)n}z), & y \text{ dipole,}\end{cases} \tag{17}$$

where L is length of the structure that is normalized to be 1 m in our computation, V_g is group velocity, and c is speed of light. As well, transverse wakefields expression from Eq. (16) is modified to be

$$\vec{F}_\perp=e\sum_{k,n}\begin{cases}|E_z|_{(2k)n}\dfrac{\sin\beta_{(2k)n}}{\beta_{(2k)n}}\left(1-\dfrac{Vg_{(2k)n}}{c-Vg_{(2k)n}}\dfrac{z}{L}\right)\left(\vec{\alpha}_x\dfrac{(2k)\pi}{w}\cos\dfrac{(2k)\pi}{w}x\cosh|k_y^{(0)}|y+\vec{\alpha}_y|k_y^{(0)}|\sin\dfrac{(2k)\pi}{w}x\sinh|k_y^{(0)}|y\right),\\ \quad x \text{ dipole}\\ |E_z|_{(2k-1)n}\dfrac{\sin\beta_{(2k-1)n}}{\beta_{(2k-1)n}}\left(1-\dfrac{Vg_{(2k-1)n}}{c-Vg_{(2k-1)n}}\dfrac{z}{L}\right)\left(\vec{\alpha}_x\dfrac{(2k-1)\pi}{w}\sin\dfrac{(2k-1)\pi}{w}\right.\\ \quad\left.\times x\sinh|k_y^{(0)}|y+\vec{\alpha}_y|k_y^{(0)}|\cos\dfrac{(2k-1)\pi}{w}x\cosh|k_y^{(0)}|y\right), \quad y \text{ dipole.}\end{cases} \tag{18}$$

TABLE I. Parameters of x-dipole synchronous accelerating modes excited by a charge traveling at transverse offset distance $x_0=1$ mm and $y_0=0$.

Mode	Freq. (GHz)	β_i (rad/m)	$(R/Q)_i$	V_g/c
LSM_{21}^{open}	11.95	250.413	313.224	0.121
LSE_{21}^{open}	14.77	309.617	268.956	0.235
LSM_{41}^{open}	14.43	302.405	99.143	0.109
LSE_{41}^{open}	18.36	384.701	269.121	0.16
LSM_{61}^{open}	17.57	368.282	17.383	0.104
LSE_{61}^{open}	21.84	457.697	86.92	0.13
LSM_{22}^{open}	33.92	710.998	171.053	0.237

TABLE II. Parameters of y-dipole synchronous accelerating modes excited by a charge traveling at transverse offset $x_0=0$ mm and $y_0=1$ mm.

Mode	Freq. (GHz)	β_i (rad/m)	$(R/Q)_i$	V_g/c
LSM_{11}^{short}	7.319	153.392	461.556	0.472
LSE_{11}^{short}	15.44	323.512	227.808	0.162
LSM_{31}^{short}	12.72	266.685	283.758	0.142
LSE_{31}^{short}	17.33	363.274	472.109	0.148
LSM_{51}^{short}	15.9	333.305	61.771	0.108
LSE_{51}^{short}	20.27	424.755	230.643	0.133
LSM_{32}^{short}	33.79	708.15	213.547	0.335

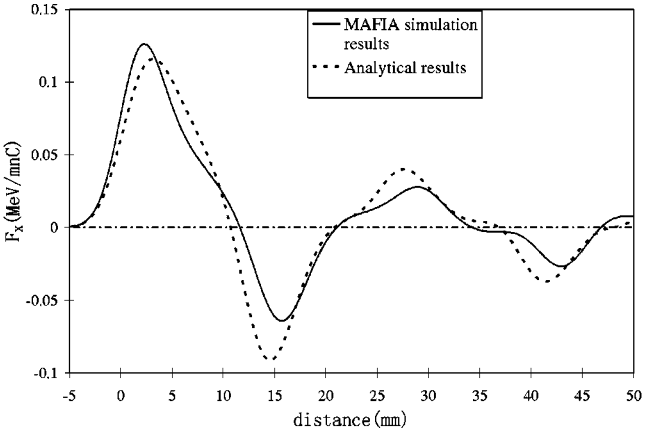

FIG. 3. Calculated and MAFIA® simulated transverse wake fields at $x_0=1$ mm and $y_0=0$ in an X-Band structure ($a=3$, $b=5$, $w=23$ mm, $\varepsilon_r=10$, and $\sigma_z=2$ mm, $q=1$ nc) due to x-dipole modes.

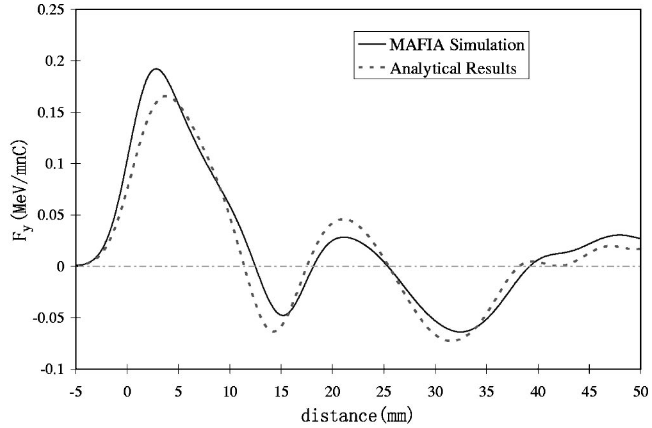

FIG. 4. Calculated and MAFIA® simulated transverse wakefields at $x_0=0$ and $y_0=1$ mm in an X-Band structure due to Y-dipole modes.

In the numerical calculation, we assume a Gaussian longitudinal beam shape (with rms bunch length σ_z and charge q). Table I shows the calculated results for leading x-dipole modes synchronous with an ultrarelativistic ($\beta=2\pi f/c$) electron bunch.

In Fig. 3, the transverse wakefields obtained using Eq. (18) are shown. For this example, a bunch with $\sigma_z=2$ mm and $q=1$ nC located at $x_0=1$ mm and $y_0=0$ is traversing along the axis at the speed of light in dielectric-loaded waveguide. Although we only list first 7 x-dipole modes, a total of 18 x-dipole modes are used for the wakefields calculations. In order to confirm the analytical method developed here, a commercial EM simulation tool MAFIA® is used for the same structure. Since MAFIA® gives total wakefields that include all the modes, such as monopole, dipole, and other higher-order modes, one must deduce the dipole wakefields from the wakefield results at both the beam position ($x_0=1$ mm, $y_0=0$) and the center ($x_0=0$ mm, $y_0=0$). A very good agreement is found and illustrated in Fig. 3. The small shape and amplitude deviations could be due to higher-order quadrapole modes (not investigated in this paper).

For the completeness, we have also calculated the corresponding results for y-dipole modes which are shown in Table II.

The transverse wakefield obtained is shown in Fig. 4. The bunch is located at $x_0=0$ and $y_0=1$ mm. The 18 y-dipole modes are also used here. It also shows a good agreement with the MAFIA® simulations.

In summary, we have presented a different approach to the analysis of dipole transverse wakefields in a dielectric-loaded rectangular waveguide accelerating structure which gives a clear understanding of each individual mode's contribution. The wakefields can be constructed as linear superposition of each LSE and LSM modes which is characterized as the geometric quantity R/Q. This approach gives a complete description of the wakefields in the partially dielectric loaded waveguide structures. The analytical results are in excellent agreement with the numerical simulations using MAFIA®.

This work was supported by Department of Energy, High Energy Physics Division, under the Contract No. W-31-109-ENG-38.

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